

## Transformations of Sin and Cos

See applet: <http://waldomaths.com/Sine1NLW.jsp>

There are four basic types of transformation of the sine and cosine curve:  
(A, B, C and D can be positive or negative)

Type 1       $\sin x \rightarrow A \sin x$  - a vertical stretch with factor A

Type 2       $\sin x \rightarrow \sin Bx$  - a horizontal stretch with factor  $\frac{1}{B}$  (a 'squeeze')

Type 3       $\sin x \rightarrow \sin(x+C)$  - a horizontal shift of  $-C$ , as a vector =  $\begin{pmatrix} -C \\ 0 \end{pmatrix}$

Type 4       $\sin x \rightarrow \sin x + D$  - a vertical shift of D, as a vector =  $\begin{pmatrix} 0 \\ D \end{pmatrix}$

The same principle also applies to the cos function (and indeed all functions)

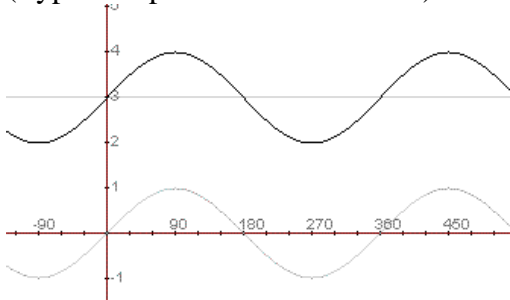
There are two other transformations derived from the above

5       $\sin x \rightarrow -\sin x$  - a reflection in the  $x$ -axis (up-down), from 2 above

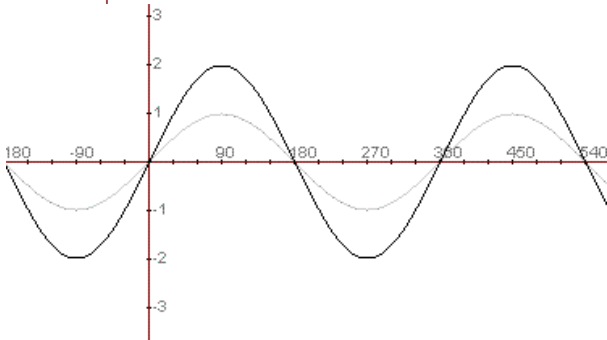
6       $\sin x \rightarrow \sin(-x)$  - a reflection in the  $y$ -axis (left-right), from 2 above

### A Single Transformations

A  $y = \sin x$  and  $y = \sin x + 3$   
(Type 4 - upward shift of 3 units)

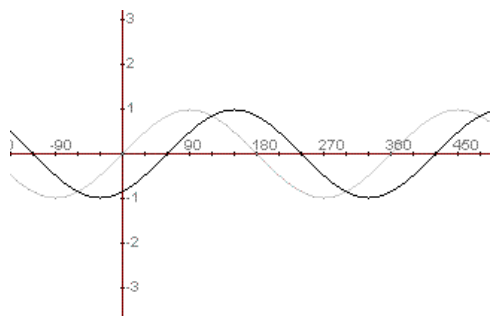


B  $y = \sin x$  and  $y = 2\sin x$   
(Type 1 - stretch parallel to  $y$ -axis, factor 2)

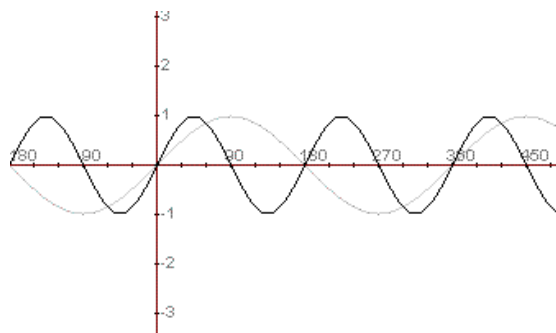


C  $y = \sin x$  and  $y = \sin(x - 60^\circ)$   
(Type 3 - shift to **right** of  $60^\circ$ )

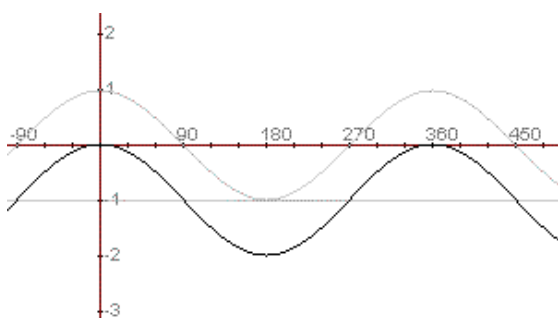
D  $y = \sin x$  and  $y = \sin 2x$   
(Type 2 - stretch parallel to  $x$ -axis, factor  $\frac{1}{2}$ )



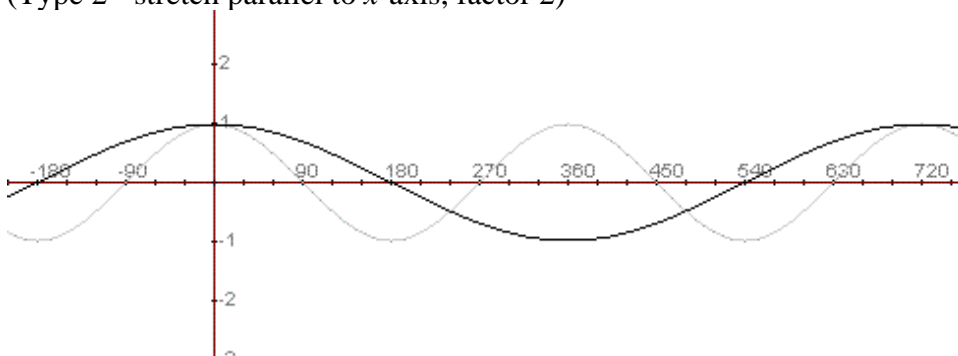
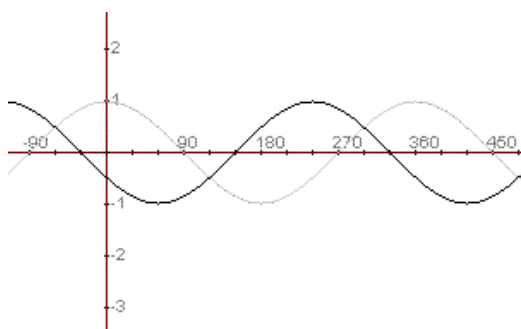
**E**  $y = \cos x$  and  $y = \cos x - 1$   
(Type 4 - shift down by 1 unit)



**F**  $y = \cos x$  and  $y = \cos(x + 120^\circ)$   
(Type 3 - shift to **left** by  $120^\circ$ )



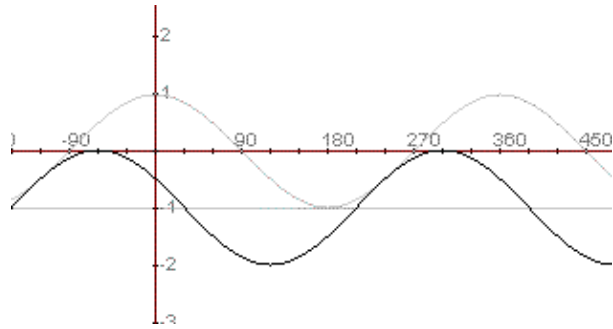
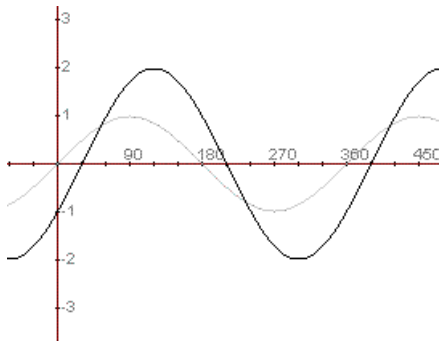
**G**  $y = \cos x$  and  $y = \cos \frac{1}{2}x$   
(Type 2 - stretch parallel to  $x$ -axis, factor 2)



## **B Multiple Transformations**

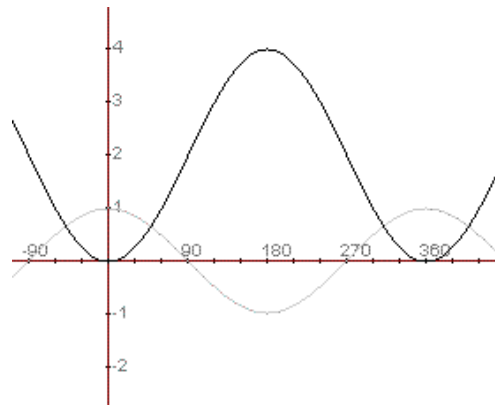
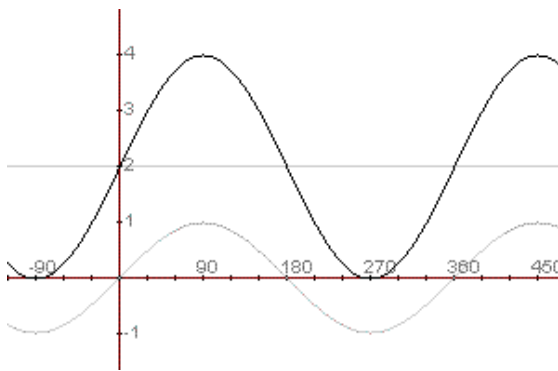
**H**  $y = \sin x$  and  $y = 2 \sin(x - 30^\circ)$   
Types 1 & 3  
(stretch up ( $\times 2$ ) + shift right by  $30^\circ$ )

**I**  $y = \cos x$  and  $y = \cos(x + 60^\circ) - 1$   
Types 3 & 4  
(shift down by 1 and left by  $60^\circ$ )



J  $y = \sin x$  and  $y = 2 \sin x + 2$   
Types 1 & 4  
(vert. stretch ( $\times 2$ ) + up shift by 2)

K  $y = \cos x$  and  $y = -2 \cos x + 2$   
Types 1 & 4  
(vert. stretch by ( $\times -2$ ) + up shift by 2)



L  $y = \sin x$  and  $y = \sin 4x - 2$   
Types 2 & 4  
(horiz. stretch ( $\times \frac{1}{4}$ ) + shift down by 2)

M  $y = \sin x$  and  $y = 3 \sin 2x + 1$   
Types 1, 2 & 4  
(horiz. stretch ( $\times \frac{1}{2}$ ) + vert. stretch ( $\times 3$ ) +  
vert. shift by 1)

