

Transformations of Sin and Cos

See applet: <http://waldomaths.com/Sine1NLW.jsp>

There are four basic types of transformation of the sine and cosine curve:
(A, B, C and D can be positive or negative)

Type 1 $\sin x \rightarrow A \sin x$ - a vertical stretch with factor A

Type 2 $\sin x \rightarrow \sin Bx$ - a horizontal stretch with factor $\frac{1}{B}$ (a 'squeeze')

Type 3 $\sin x \rightarrow \sin(x+C)$ - a horizontal shift of $-C$, as a vector = $\begin{pmatrix} -C \\ 0 \end{pmatrix}$

Type 4 $\sin x \rightarrow \sin x + D$ - a vertical shift of D, as a vector = $\begin{pmatrix} 0 \\ D \end{pmatrix}$

The same principle also applies to the cos function (and indeed all functions)

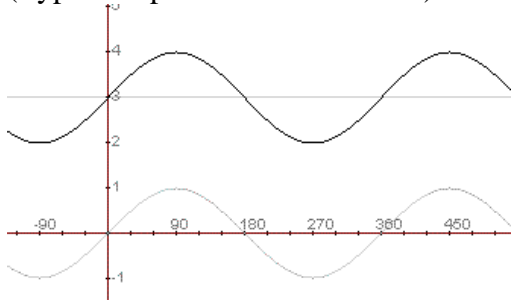
There are two other transformations derived from the above

5 $\sin x \rightarrow -\sin x$ - a reflection in the x -axis (up-down), from 2 above

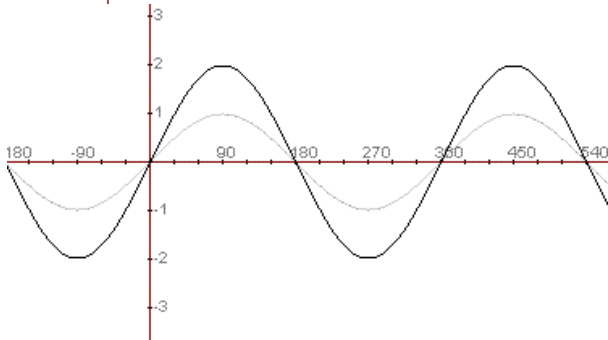
6 $\sin x \rightarrow \sin(-x)$ - a reflection in the y -axis (left-right), from 2 above

A Single Transformations

A $y = \sin x$ and $y = \sin x + 3$
(Type 4 - upward shift of 3 units)

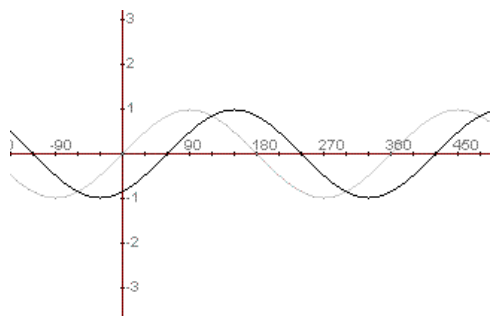


B $y = \sin x$ and $y = 2\sin x$
(Type 1 - stretch parallel to y -axis, factor 2)

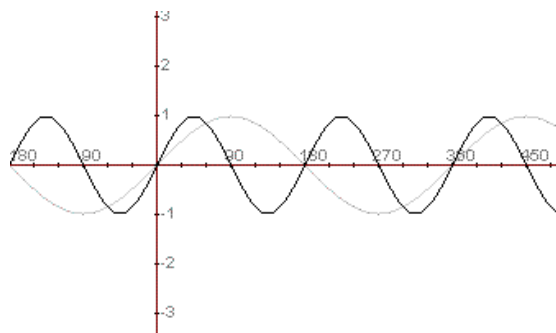


C $y = \sin x$ and $y = \sin(x - 60^\circ)$
(Type 3 - shift to **right** of 60°)

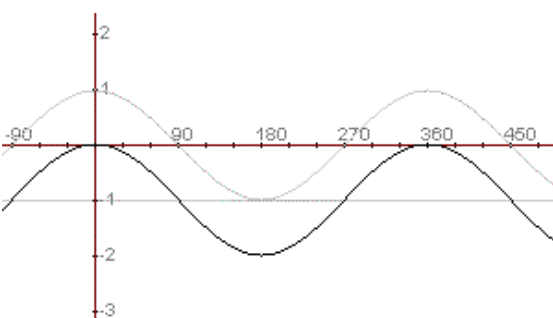
D $y = \sin x$ and $y = \sin 2x$
(Type 2 - stretch parallel to x -axis, factor $\frac{1}{2}$)



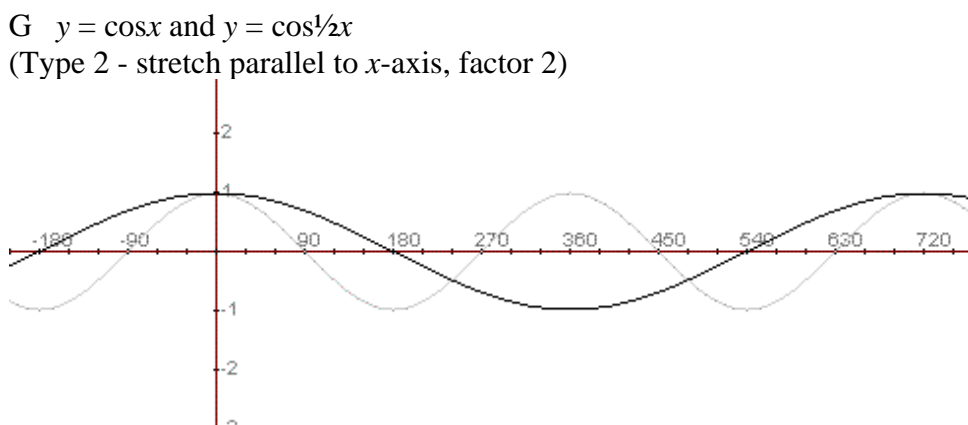
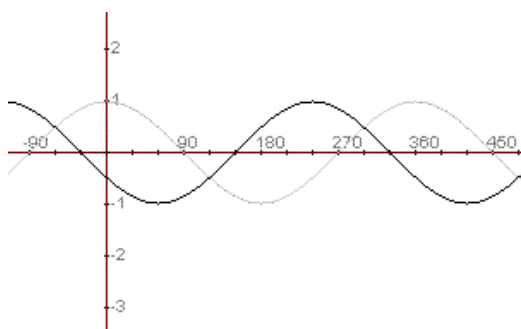
E $y = \cos x$ and $y = \cos x - 1$
(Type 4 - shift down by 1 unit)



F $y = \cos x$ and $y = \cos(x + 120^\circ)$
(Type 3 - shift to **left** by 120°)



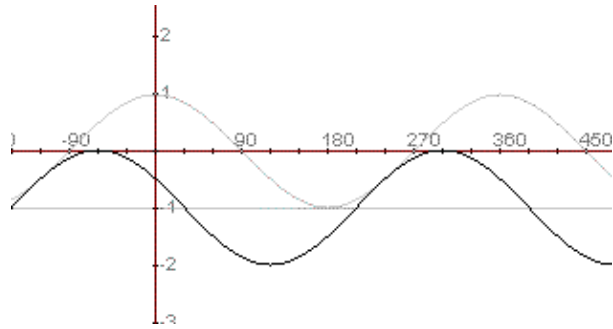
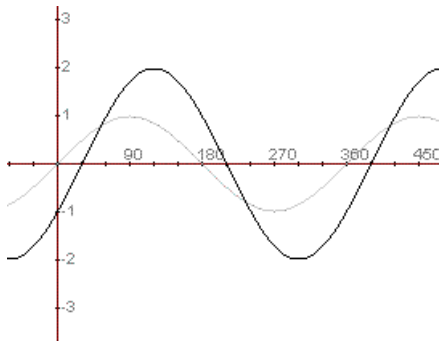
G $y = \cos x$ and $y = \cos \frac{1}{2}x$
(Type 2 - stretch parallel to x -axis, factor 2)



B Multiple Transformations

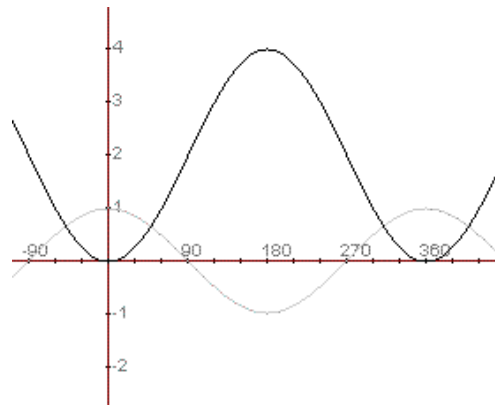
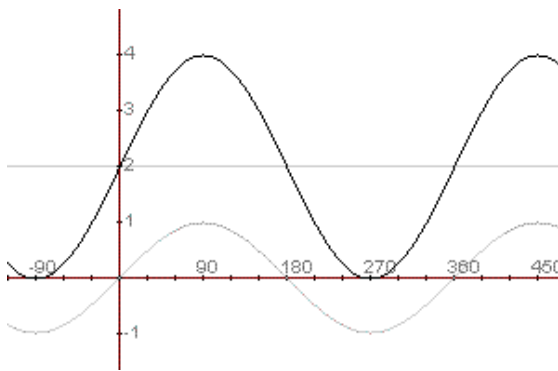
H $y = \sin x$ and $y = 2 \sin(x - 30^\circ)$
Types 1 & 3
(stretch up ($\times 2$) + shift right by 30°)

I $y = \cos x$ and $y = \cos(x + 60^\circ) - 1$
Types 3 & 4
(shift down by 1 and left by 60°)



J $y = \sin x$ and $y = 2 \sin x + 2$
 Types 1 & 4
 (vert. stretch ($\times 2$) + up shift by 2)

K $y = \cos x$ and $y = -2 \cos x + 2$
 Types 1 & 4
 (vert. stretch by ($\times -2$) + up shift by 2)



L $y = \sin x$ and $y = \sin 4x - 2$
 Types 2 & 4
 (horiz. stretch ($\times \frac{1}{4}$) + shift down by 2)

M $y = \sin x$ and $y = 3 \sin 2x + 1$
 Types 1, 2 & 4
 (horiz. stretch ($\times \frac{1}{2}$) + vert. stretch ($\times 3$) +
 vert. shift by 1)

