Difference of Two Squares

See applet: http://www.waldomaths.com/FactorQuad2NL.jsp & check the "DOTS" box

Whenever an algebraic expression is of the form $a^2 - b^2$, it can be factorised as:

$$a^2 - b^2 = (a+b)(a-b)$$

This is known as the **difference of two squares (D.O.T.S)**.
[NB the technique cannot be used for $a^2 + b^2$, which cannot be factorised.
Neither can $-a^2 - b^2$, which is simply $-(a^2 + b^2)$.]

<u>Examples</u>:

(1) Factorise $x^2 - y^2$ <u>Answer</u>: $(x+y)(x-y)$ or $(x-y)(x+y)$

(2) Factorise $4p^2 - 25q^2$ <u>Answer</u>: $(2p)^2 - (5q)^2 = (2p+5q)(2p-5q)$

(3) Factorise $9a^2x^2 - 16b^2y^2$

<u>Answer</u>: $(3ax)^2 - (4by)^2 = (3ax+4by)(3ax-4by)$

Notice that in examples (2) and (3) it is best to write each of the two terms as a single squared term first.

(4) Factorise $81x^4 - 121y^{10}$

<u>Answer:</u> $(9x^2)^2 - (11y^5)^2 = (9x^2 + 11y^5)(9x^2 - 11y^5)$

(5) Factorise $(a+b)^2 - (a-b)^2$

<u>Answer:</u> $\{(a+b)-(a-b)\}\{(a+b)+(a-b)\} = (2a)(2b) = 4ab$

[NB this answer is still 'factorised' as it is different quantities multiplied together.]

Important: this technique can only be used when the two terms can be written as single squares, and there is a minus sign between them. It is about the **difference** of two squares.

Questions: Factorise:

(a) $m^2 - n^2$ (b) $49s^2 - t^2$ (c) $100c^2 - 144d^2$ (d) $(ab)^2 - (pq)^2$

(e) $4m^2n^2 - p^2q^2$ (f) $81e^4 - 36f^2$ (g) $64a^2b^2c^2 - 144d^2e^2f^2$

(h) $x^4 - y^4$ (take care!) (i) $(2a+1)^2 - (2a-1)^2$ (j) $(2x+1)^2 - (2y-1)^2$

Answers: (a) $(m+n)(m-n)$ (b) $(7s+t)(7s-t)$ (c) $4(5c+6d)(5c-6d)$

(d) $(ab-pq)(ab+pq)$ (e) $(2mn-pq)(2mn+pq)$ (f) $9(3e^2+2f)(3e^2-2f)$

(g) $16(2abc-3def)(2abc+3def)$ (h) $(x^2+y^2)(x+y)(x-y)$

(i) $8a$ (j) $4(x-y-1)(x+y)$

Copyright © R F Barrow 2007