5.13 A2 MODULE 4733: PROBABILITY AND STATISTICS 2 (S2)

Preamble

Knowledge of the specification content of Modules *C1* to *C4* and *S1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *S2*.

Continuous Random Variables

Candidates should be able to:

(a) understand the concept of a continuous random variable, and recall and use properties of a probability density function (restricted to functions defined over a single interval);

(b) use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution (explicit knowledge of the cumulative distribution function is not included, but location of the median, for example, in simple cases by direct consideration of an area may be required).

The Normal Distribution

Candidates should be able to:

(a) understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables, or equivalent calculator functions (knowledge of the density function is not expected);

(b) solve problems concerning a variable *X*, where $X \sim \mathrm{N}(\mu, \sigma^2)$, including

  (i) finding the value of $\mathrm{P}(X > x_1)$, or a related probability, given the values of x_1, μ, σ,

  (ii) finding a relationship between x_1, μ and σ given the value of $\mathrm{P}(X > x_1)$ or a related probability;

(c) recall conditions under which the normal distribution can be used as an approximation to the binomial distribution (*n* large enough to ensure that $np > 5$ and $nq > 5$), and use this approximation, with a continuity correction, in solving problems.

The Poisson Distribution

Candidates should be able to:

(a) calculate probabilities for the distribution $\mathrm{Po}(\mu)$, both directly from the formula and also by using tables of cumulative Poisson probabilities (or equivalent calculator functions);

(b) use the result that if $X \sim \mathrm{Po}(\mu)$ then the mean and variance of *X* are each equal to μ;

(c) understand informally the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model;

(d) use the Poisson distribution as an approximation to the binomial distribution where appropriate ($n > 50$ and $np < 5$, approximately);

(e) use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate ($\mu > 15$, approximately).

Sampling and Hypothesis Tests

IT3.1

Candidates should be able to:

(a) understand the distinction between a sample and a population, and appreciate the benefits of randomness in choosing samples;

(b) explain in simple terms why a given sampling method may be unsatisfactory and suggest possible improvements (knowledge of particular methods of sampling, such as quota or stratified sampling, is not required, but candidates should have an elementary understanding of the use of random numbers in producing random samples);

(c) recognise that a sample mean can be regarded as a random variable, and use the facts that $\mathrm{E}(\bar{X}) = \mu$ and that $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$;

(d) use the fact that $\bar{X}$ has a normal distribution if X has a normal distribution;

(e) use the Central Limit Theorem where appropriate;

(f) calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data (only a simple understanding of the term ‘unbiased’ is required);

(g) understand the nature of a hypothesis test, the difference between one-tail and two-tail tests, and the terms ‘null hypothesis’, ‘alternative hypothesis’, ‘significance level’, ‘rejection region’ (or ‘critical region’), ‘acceptance region’ and ‘test statistic’;

(h) formulate hypotheses and carry out a hypothesis test of a population proportion in the context of a single observation from a binomial distribution, using either direct evaluation of binomial probabilities or a normal approximation with continuity correction;

(i) formulate hypotheses and carry out a hypothesis test of a population mean in the following cases:

 (i) a sample drawn from a normal distribution of known variance,

 (ii) a large sample, using the Central Limit Theorem and an unbiased variance estimate derived from the sample,

 (iii) a single observation drawn from a Poisson distribution, using direct evaluation of Poisson probabilities;

(j) understand the terms ‘Type I error’ and ‘Type II error’ in relation to hypothesis tests;

(k) calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or approximation, or on direct evaluation of binomial or Poisson probabilities.