5.5 AS MODULE 4725: FURTHER PURE MATHEMATICS 1 (FP1)

Preamble

Knowledge of the specification content of Modules *C1* and *C2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *FP1*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Algebra

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

For $ax^2 + bx + c = 0$: $\alpha + \beta = -\dfrac{b}{a}, \quad \alpha\beta = \dfrac{c}{a}$

For $ax^3 + bx^2 + cx + d = 0$: $\Sigma\alpha = -\dfrac{b}{a}, \quad \Sigma\alpha\beta = \dfrac{c}{a}, \quad \alpha\beta\gamma = -\dfrac{d}{a}$

Matrices

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Summation of Series

Candidates should be able to:

(a) use the standard results for Σr , Σr^2 , Σr^3 to find related sums;

(b) use the method of differences to obtain the sum of a finite series;

(c) recognise, by direct consideration of the sum to *n* terms, when a series is convergent, and find the sum to infinity in such cases.

Proof by Induction

Candidates should be able to:

(a) use the method of mathematical induction to establish a given result (not restricted to summation of series);

(b) recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. to find the *n*th power of the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

Roots of Polynomial Equations

Candidates should be able to:

(a) use the relations between the symmetric functions of the roots of polynomial equations and the coefficients (for equations of degree 2 or 3 only);

(b) use a given simple substitution to obtain an equation whose roots are related in a simple way to those of the original equation.

Complex Numbers

Candidates should be able to:

(a) understand the idea of a complex number, recall the meaning of the terms ‘real part’, ‘imaginary part’, ‘modulus’, ‘argument’, ‘conjugate’, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal;

(b) carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form $(x + \mathrm{i}\,y)$;

(c) use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs;

(d) represent complex numbers geometrically by means of an Argand diagram, and understand the geometrical effects of conjugating a complex number and of adding and subtracting two complex numbers;

(e) find the two square roots of a complex number;

(f) illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, e.g. $|z-a| < k$, $|z-a| = |z-b|$, $\arg(z-a) = \alpha$.

Matrices

Candidates should be able to:

(a) carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix;

(b) recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices, and, for 2×2 and 3×3 matrices, evaluate determinants and find inverses of non-singular matrices;

(c) understand and use the result, for non-singular matrices, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$;

(d) understand the use of 2×2 matrices to represent certain geometrical transformations in the *x*-*y* plane, and in particular

(i) recognise that the matrix product **AB** represents the transformation that results from the transformation represented by **B** followed by the transformation represented by **A**,

(ii) recall how the area scale-factor of a transformation is related to the determinant of the corresponding matrix,

(iii) find the matrix that represents a given transformation or sequence of transformations (understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' will be required);

(e) formulate a problem involving the solution of 2 linear simultaneous equations in 2 unknowns, or 3 equations in 3 unknowns, as a problem involving the solution of a matrix equation, and vice-versa;

(f) understand the cases that may arise concerning the consistency or inconsistency of 2 or 3 linear simultaneous equations, relate them to the singularity or otherwise of the corresponding square matrix, and solve consistent systems.