5.1 AS MODULE 4721: CORE MATHEMATICS 1 (C1)

Preamble

No calculators are permitted in the assessment of this unit.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Algebra

Solution of $ax^2 + bx + c = 0$ is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant of $ax^2 + bx + c$ is $b^2 - 4ac$

Coordinate Geometry

Equation of the straight line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Equation of the circle with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$

Differentiation

If $y = x^n$ then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = f(x) + g(x)$ then $\dfrac{dy}{dx} = f'(x) + g'(x)$

Indices and Surds

Candidates should be able to:

(a) understand rational indices (positive, negative and zero), and use laws of indices in the course of algebraic applications;

(b) recognise the equivalence of surd and index notation (e.g. $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a^2} = a^{\frac{2}{3}}$);

(c) use simple properties of surds such as $\sqrt{12} = 2\sqrt{3}$, including rationalising denominators of the form $a + \sqrt{b}$.

Polynomials

Candidates should be able to:

(a) carry out operations of addition, subtraction, and multiplication of polynomials (including expansion of brackets, collection of like terms and simplifying);

(b) carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$, and use this form, e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$;

(c) find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant, e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$;

(d) solve quadratic equations, and linear and quadratic inequalities, in one unknown;

(e) solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic;

(f) recognise and solve equations in *x* which are quadratic in some function of *x*, e.g. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4 = 0$.

Coordinate Geometry and Graphs

Candidates should be able to:

(a) find the length, gradient and mid-point of a line-segment, given the coordinates of its end-points;

(b) find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it, or one point on it and its gradient);

(c) understand and use the relationships between the gradients of parallel and perpendicular lines;

(d) interpret and use linear equations, particularly the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$;

(e) understand that the equation $(x-a)^2 + (y-b)^2 = r^2$ represents the circle with centre (a, b) and radius *r*;

(f) use algebraic methods to solve problems involving lines and circles, including the use of the equation of a circle in expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$ (knowledge of the following circle properties is included: the angle in a semicircle is a right angle; the perpendicular from the centre to a chord bisects the chord; the perpendicularity of radius and tangent);

(g) understand the relationship between a graph and its associated algebraic equation, use points of intersection of graphs to solve equations, and interpret geometrically the algebraic solution of equations (to include, in simple cases, understanding of the correspondence between a line being tangent to a curve and a repeated root of an equation);

(h) sketch curves with equations of the form

(i) $y = kx^n$, where *n* is a positive or negative integer and *k* is a constant,

(ii) $y = k\sqrt{x}$, where *k* is a constant,

(iii) $y = ax^2 + bx + c$, where *a*, *b* and *c* are constants,

(iv) $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the product of at most 3 linear factors, not necessarily all distinct;

(i) understand and use the relationships between the graphs of $y = \mathrm{f}(x)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$, where *a* is a constant, and express the transformations involved in terms of translations, reflections and stretches.

Differentiation

Candidates should be able to:

(a) understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords (an informal understanding only is required, and the technique of differentiation from first principles is not included);

(b) understand the ideas of a derived function and second order derivative, and use the notations $\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$;

(c) use the derivative of x^n (for any rational *n*), together with constant multiples, sums and differences;

(d) apply differentiation (including applications to practical problems) to gradients, tangents and normals, rates of change, increasing and decreasing functions, and the location of stationary points (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included).