GCSE Notes and Revision – Surds

See videos : http://www.waldomaths.com/video/Surds01/Surds01.jsp + +

Rules of surds

(1) Multiplication $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$, eg. $\sqrt{2}\times\sqrt{3}=\sqrt{6}$

(2) Division $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$, eg. $\frac{\sqrt{10}}{\sqrt{2}}=\sqrt{\frac{10}{2}}=\sqrt{5}$

(3) Squaring and rooting $(\sqrt{a})^2=a$, eg. $\sqrt{7}\times\sqrt{7}=(\sqrt{7})^2=7$

$\sqrt{a^2}=a$, eg. $\sqrt{36}=\sqrt{6^2}=6$

Simplifying surd expressions

(A) It is normal to make the number inside the square root sign as small as possible

Example 1: Simplify $\sqrt{12}$.

Answer: $\sqrt{12}=\sqrt{4\times3}=\sqrt{4}\times\sqrt{3}=2\times\sqrt{3}=2\sqrt{3}$ (In other words, factorise the number and take out any factors which are **square numbers** and square root them. The number left in the square root sign has no more square factors)

Example 2: Simplify $\sqrt{108}$.

Answer: $\sqrt{108}=\sqrt{9}\times\sqrt{12}=\sqrt{9}\times\sqrt{4}\times\sqrt{3}=3\times2\times\sqrt{3}=6\sqrt{3}$

Questions 1: Simplify: (a) $\sqrt{18}$ (b) $\sqrt{48}$ (c) $\sqrt{50}$ (d) $\sqrt{84}$
(e) $\sqrt{90}$ (f) $\sqrt{98}$ (g) $\sqrt{132}$ (h) $\sqrt{200}$ (i) $\sqrt{52}$

(B) You can simplify multiples to a single surd expression

Example 1: Simplify $\sqrt{2}\times\sqrt{3}$ Answer: $\sqrt{6}$

Example 2: Simplify $\sqrt{6}\times\sqrt{10}$ Answer: $\sqrt{6}\times\sqrt{10}=\sqrt{60}=\sqrt{4}\times\sqrt{15}=2\sqrt{15}$

Example 3: Simplify $\sqrt{26}\times\sqrt{39}$ Answer: $\sqrt{2\times13\times3\times13}=\sqrt{6\times13^2}=13\sqrt{6}$

Questions 2: Simplify: (a) $\sqrt{6}\times\sqrt{15}$ (b) $\sqrt{10}\times\sqrt{15}$ (c) $\sqrt{14}\times\sqrt{21}$

(C) You can add or subtract expressions if they are multiples of the same surd

Example 1: $3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$

Example 2: $\sqrt{18}-\sqrt{8}=3\sqrt{2}-2\sqrt{2}=\sqrt{2}$

Questions 3: Simplify: (a) $\sqrt{40}+\sqrt{10}$ (b) $\sqrt{27}+\sqrt{12}$ (c) $\sqrt{24}-\sqrt{6}$
(d) $\sqrt{125}-\sqrt{75}$ (e) $\sqrt{44}+\sqrt{99}$

(D) It is usual to “rationalise the denominator”. In other words, if there is a surd in the denominator of a fraction, simplify so that the only surds appear in the numerator.

Example 1: Simplify $\frac{2}{\sqrt{3}}$ Answer: $\frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$ (notice that although the answer seems less simple, there are no surds in the denominator)

Example 2: Simplify $\frac{21}{\sqrt{14}}$ Answer: $\frac{21}{\sqrt{14}}\times\frac{\sqrt{14}}{\sqrt{14}}=\frac{21\sqrt{14}}{14}=\frac{3\sqrt{14}}{2}$

Questions 4: Simplify: (a) $\frac{10}{\sqrt{5}}$ (b) $\frac{\sqrt{5}}{\sqrt{3}}$ (c) $\frac{2\sqrt{3}+5}{\sqrt{3}}$

(d) $\frac{3\sqrt{6}}{\sqrt{3}}$ (e) $\frac{12\sqrt{21}}{\sqrt{6}}$ (f) $\frac{2\sqrt{15}}{\sqrt{12}}$ (g) $\frac{12\sqrt{3}-6\sqrt{2}}{\sqrt{6}}$

(E) Multiplying brackets with surd expressions. Initially you should treat the surd expression like an algebraic expression, and simplify after multiplying the brackets.

Example 1: Simplify $(\sqrt{3}-2\sqrt{2})(2\sqrt{3}-\sqrt{2})$

Answer: $2(\sqrt{3})^2-4\sqrt{2}\sqrt{3}-\sqrt{2}\sqrt{3}+2(\sqrt{2})^2 = 10-5\sqrt{6}$

Example 2: Simplify $(\sqrt{5}-\sqrt{3})^2$

Answer: $(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})=(\sqrt{5})^2-\sqrt{3}\sqrt{5}-\sqrt{3}\sqrt{5}-(\sqrt{3})^2$

$= 8-2\sqrt{15}$

Example 3: Simplify $(3\sqrt{7}-2\sqrt{5})(3\sqrt{7}+2\sqrt{5})$

Answer: $(3\sqrt{7})^2-6\sqrt{35}+6\sqrt{35}-(2\sqrt{5})^2 = 63-20=43$

Questions 5: Expand brackets and simplify (a) $(\sqrt{5}-2)(2\sqrt{5-1})$

(b) $(3\sqrt{5}-\sqrt{7})^2$ (c) $(2\sqrt{11}-3\sqrt{6})(2\sqrt{11}+3\sqrt{6})$

General Questions 6:

(a) Show that $x=1+\sqrt{5}$ is a solution of the quadratic equation $x^2-2x-4=0$.

(b) A right-angled triangle has the two shorter sides $\sqrt{3}-1$ and $\sqrt{3}+1$. Show that the hypotenuse has length $2\sqrt{2}$.

(c) A rectangle has two sides $\sqrt{7}-1$ and $x\sqrt{7}+2$. Its area is 14 square units. Show that $x=2$.

(d) Show that the positive square root of $(7+4\sqrt{3})$ is $(\sqrt{3}+2)$.

ANSWERS:

1	(a)	$3\sqrt{2}$	(b)	$4\sqrt{3}$	(c)	$5\sqrt{2}$	(d)	$2\sqrt{21}$
	(e)	$3\sqrt{10}$	(f)	$7\sqrt{2}$	(g)	$2\sqrt{33}$	(h)	$2\sqrt{13}$
2	(a)	$3\sqrt{10}$	(b)	$5\sqrt{6}$	(c)	$7\sqrt{6}$		
3	(a)	$3\sqrt{10}$	(b)	$5\sqrt{3}$	(c)	$\sqrt{6}$	(d)	$5(\sqrt{5}-\sqrt{3})$
	(e)	$5\sqrt{11}$						
4	(a)	$2\sqrt{5}$	(b)	$\frac{\sqrt{15}}{3}$	(c)	$\frac{(2\sqrt{3}+5)\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{6+5\sqrt{3}}{3} or 2+\frac{5\sqrt{3}}{3}$		
	(d)	$3\sqrt{2}$	(e)	$6\sqrt{14}$	(f)	$\sqrt{5}$	(g)	$6\sqrt{2}-2\sqrt{3}$
5.	(a)	$8-5\sqrt{5}$	(b)	$52-6\sqrt{35}$	(c)	-10		